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ORIENTATIONAL DYNAMICS AND ENERGY DISSIPATION IN A LIQUID DISPERSION OF SINGLE-DOMAIN FERROPARTICLES ON EXPOSURE TO A LINEARLY POLARIZED FIELD

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A theoretical investigation of the nonlinear orientational dynamics and energy dissipation in a suspension of noninteracting single-domain ferroparticles with a magnetic hysteresis on exposure to a linearly polarized magnetic field has been performed. The bifurcation properties of the system have been studied; the amplitude and frequency dependences of the powers of total and viscous dissipation have been obtained.

Introduction. The possibility of using heat releases in a ferromagnetic material with a hysteresis magnetization reversal as the volume heat source in problems of heating, primarily of dielectric bodies, was studied for the first time in [1]. Particularly promising are small ferromagnetic particles using which one can form volume heaters of adaptable shape and size, up to the cell size, which seems pressing in connection with the trend toward miniaturization and with the development of nano-size technologies. In the present work, in addition to the case (investigated earlier [1]) of a solid dispersion, we consider dissipative phenomena in a liquid dispersion of single-domain ferroparticles that is exposed to a linearly polarized variable magnetic field.

The fundamental difference of a liquid dispersion from a solid one is the mechanical mobility of particles, which creates the competitive mechanism of orientational relaxation and an additional (viscous) source of energy dissipation. The foundation of the investigation proposed is a theoretical analysis of the joint mechanical and magnetic orientational dynamics of a single-domain particle. This problem has been considered for the first time in the entire range of excitation parameters. A restriction (of no practical importance, however) is imposed only on the period of variation of the field, which is assumed to be longer than the time (measured in nanoseconds) of orientational relaxation of the magnetic moment in the particle's solid matrix.

It is noteworthy that the orientational behavior of single-domain ferrosuspension particles in variable fields in magnetization hysteresis was considered earlier, too [2–4]. It has been found that the character of the orientational dynamics excited by the external field depends on the values of two dimensionless parameters. One parameter, a_0, is the ratio of the amplitude of the external magnetic field to the effective field of magnetic anisotropy of a particle, whereas the other, ν, is the product of the field frequency by the characteristic time of mechanical rotation of a particle in a viscous fluid. We were able to obtain the analytical solution of the problem for arbitrary values of these parameters only for the case of a uniformly rotating (cylindrically polarized) field of constant value [2, 3]. In a linearly polarized field, the orientational behavior of a ferrosuspension has been considered in [4]. The analytical solution of this problem is available only in small ($a_0 << 1$) and large ($a_0 >> 1$) fields, whereas a numerical investigation has been performed only in the high-frequency limit ($\nu >> 1$), when the mechanical dynamics of particles is blocked by viscous forces and a certain field-amplitude-dependent stationary mechanical orientation is established in the system.

Equations of Orientational Dynamics and Energy Dissipation. We consider a system of noninteracting hard-magnetic ferroparticles suspended in a viscous fluid. The particles are assumed to be fairly small so as to ensure a homogeneous (single-domain) state of their magnetization [5]. The critical single-domain radius is tens of nanometers

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for spherical particles of different ferromagnetics; the single-domain state persists down to submicron size in needle-shaped, high-coercivity particles [6]. Also, it is assumed that the magnetic anisotropy of particles is uniaxial in character (in a needle-shaped particle, the easy-magnetization axis coincides with the long axis of the particle), and their hysteresis magnetization reversal is by coherent rotation. In this case, the orientational state of a particle is totally characterized by the unit vectors of the directions of the anisotropy axis **n** and the magnetic moment **e**. We prescribe the time dependence of the field intensity in the form $\mathbf{H}(t) = H_0\mathbf{h} \sin \omega t$, where **h** is the unit vector in the direction of polarization. Using the energy barrier KV separating the opposite directions of the anisotropy axis as the energy scale, we write the expression for the particle's orientational energy in dimensionless form:

$$u = U/KV = -a(t)(\mathbf{eh}) - (\mathbf{en})^2 . \tag{1}$$

Here $a(t) = a_0 \sin \omega t$, $a_0 = 2H_0/H_a$, and $H_a = 2K/I$ is the intensity of the effective field of magnetic anisotropy of the particle. The equilibrium position of the magnetic moment is in the plane formed by the vectors **n** and **h** [5]. With the above restriction on the field frequency, the orientational equilibrium in the magnetic variable is preserved at each instant of time under dynamic conditions, too. The fact that all three vectors **e**, **h**, and **n** are in one plane makes it possible to describe the state of the particle by the angles formed by the anisotropy axis with the direction of polarization (θ) and with the direction of the magnetic moment (φ). Relation (1) takes the form

$$u = -a(t)\cos(\theta - \varphi) - \cos^2 \varphi . \tag{2}$$

The condition of equilibrium of the magnetic moment $\partial u/\partial\varphi = 0$ leads to the equation

$$a(t)\sin(\theta - \varphi) = \sin 2\varphi . \tag{3}$$

The behavior of the roots of Eq. (3) has comprehensively been studied in the theory of magnetization curves of ferromagnetics [5]. We give here some of the results that are important for an understanding of the problem in question. It is noteworthy, in particular, that, in the zero field, Eq. (3) has two stable solutions ($\varphi = 0$ and π) corresponding to the metastable (separated by the energy barrier KV) orientations of the magnetic moment in one direction of the anisotropy axis or another. In the limit of strong fields ($a \to \infty$), there is the unique thermodynamically stable solution $\varphi = \theta$ corresponding to the orientation of the moment along the direction of the field. The depth of the energy well in the direction of the field increases in application of a magnetic field of finite value along the anisotropy axis and decreases in the opposite direction. When the field intensity attains a certain critical value, one energy minimum disappears, the magnetic moment changes its metastable state to a stable one, and its orientation becomes opposite. It takes a finite time to establish a new equilibrium; this time has a value of the order of the characteristic time of attenuation of the Larmor precession in the effective magnetic field. Taking into account that this time ($\sim 10^{-9}$ sec) is negligible as compared to both the characteristic times of mechanical rotation of a particle in the viscous fluid and the period of variation of the field, we may consider the change of state of the magnetic moment to be instantaneous. It is important that the process of stepwise magnetization reversal be dissipative in character and involve the release, in the particle's body, of energy equal to the difference of the energies of the particle in the final and initial states. The critical field intensity for which the change of state occurs depends on the mutual orientation of the field and the particle's anisotropy axis. This dependence is found by elimination of the angle φ in the system of equations that includes the equilibrium equation (3) and the equation $\partial^2 u/\partial\varphi^2 = 0$ expressing the condition of the bend of the $u(\varphi)$ curve, which occurs in merging of one energy maximum with one minimum. It has the form

$$a_c^2 = 4\left[\sin^{2/3}\theta + \cos^{2/3}\theta\right]^{-3} . \tag{4}$$

The solutions of (4) are found in the interval $1 \le |a_c| \le 2$ (see [1]). The critical field has its minimum value ($|a_c| = 1$) for the orientations of the anisotropy axis $\theta = \pi/4$ and $3\pi/4$ and its maximum value for $\theta = 0$, π, and $\pi/2$. The energy jump attains its limiting value $\Delta U = KV$ with the collinear arrangement of the field and the anisotropy axis and vanishes when the arrangement is lateral. The position of the magnetic moment before the jump is found by elimination of the quantity a from the system of the equations of equilibrium and bend:

$$\tan^3 \varphi_1 = -\operatorname{tg} \theta . \tag{5}$$

The final position of the moment is determined by the equilibrium equation (3) for prescribed values of θ and $a = a_c(\theta)$ from (4). We were unable to obtain the dependence $\varphi_2(\theta)$ in explicit form.

The mechanical motion of a particle in the viscous fluid is excited by the electrodynamic moment of forces $IVH(t) \sin(\theta - \varphi)$. Disregarding the inertia of the particle, we find the equation of motion from the condition of mutual compensation of the electrodynamic moment of forces and the moment of viscous forces $6V\eta\alpha\dot{\theta}$. Introducing the time scale $t^* = 6\eta\alpha/K$, we obtain the equation sought in dimensionless form ($\dot{\theta} = d\theta/\partial\tau$):

$$\dot{\theta} = -a(\tau) \sin(\theta - \varphi) . \tag{6}$$

For a particle in the shape of an ellipsoid of revolution the shape factor α is determined by the ratio of the long and short semiaxes λ. In the case of strongly extended particles we have $\alpha = \pi\lambda^2/\ln \lambda$. Equation (6) and the condition of local magnetic equilibrium (3) make up a closed system for description of the time evolution of mechanical and magnetic orientations of a particle.

To compute the energy dissipation on a particle we use the thermodynamic relation for the work done "on the field" by the external electromotive force that is the source of currents creating the field [5]:

$$\delta R = \frac{1}{4\pi} \int \mathbf{H} \delta \mathbf{B} . \tag{7}$$

In this relation, integration is over the entire space and $\mathbf{B} = \mathbf{H} + 4\pi\mathbf{M}$, $\mathbf{M}$ being the magnetization of the dispersion. Expression (7) includes work on both creating the field $\mathbf{H}\delta\mathbf{H}$ (for example, work on maintaining the current in a solenoid) and magnetizing the suspension. In the considered case of a diluted dispersion the magnetic field in the medium can be identified with an unperturbed field created by the source. Then, for a single particle, from (7) we obtain

$$\delta R_1 = \mathbf{H}\delta\mathbf{m} \equiv IVH(t)\, \delta(\mathbf{eh}) \equiv IVH(t)\, \delta[\cos(\theta - \varphi)] . \tag{8}$$

In the stationary regime of motion, the state of the particle is repeated every period of variation of the field; we can compute the energy-dissipation power, integrating (8) in one cycle of motion and dividing the result by a time equal to the period. Moreover, in the situation in question, the energy dissipation is the same in each half-period of variation of the field, and we can restrict ourselves to a half-period in computations. The motion over a half-period may include one jump of the magnetic moment. Selecting the initial integration time to be zero and denoting the instant of time at which the jump occurs by t_j, we represent the relation for the dissipation power in the form

$$W_1 = -\frac{IV\omega H_0}{\pi} \left[\int_0^{t_j} \sin(\omega t) \sin(\theta - \varphi) \left(\frac{d\theta}{dt} - \frac{d\varphi}{dt} \right) dt + \int_{t_j}^{\pi/\omega} \sin(\omega t) \sin(\theta - \varphi) \left(\frac{d\theta}{dt} - \frac{d\varphi}{dt} \right) dt - \right.$$

$$\left. - \cos(\theta_j - \varphi_{j2}) + \cos(\theta_j - \varphi_{j1}) \right] . \tag{9}$$

Here $\theta_j = \theta(t_j)$ is the orientation of the particle at the instant of jump and φ_{j1} and φ_{j2} are the initial and final positions of the magnetic moment.

We transform (9) as follows. The terms with $d\varphi/dt$ under the integral in (9) are reduced to the form $a_0^{-1} d(\cos^2 \varphi)$ using (3). Then, according to (6), we obtain $\sin(\omega t) \sin(\theta - \varphi) = -(6\eta\alpha/IH_0)(d\theta/dt)$. Using these relations and introducing the dissipation-rate scale $W^* = KV/t^*$ and the notation $w = W/W^*$, we arrive at the following result:

$$w_1 = w_1^{\eta} + w_1^{m} , \quad w_1^{\eta} = \frac{\nu}{\pi} \int_0^{\pi/\nu} \dot{\theta}^2 \, d\tau ,$$

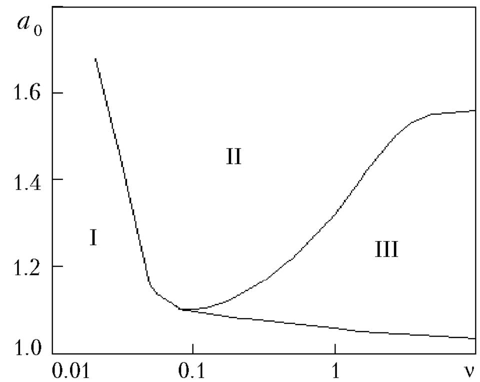


Fig. 1. Bifurcation diagram of stationary regimes of motion of a particle on the plane of dimensionless excitation parameters.

$$w_1^{\mathrm{m}} = \frac{\nu}{\pi}\left[\cos^2\varphi_{\mathrm{j1}} - \cos^2\varphi_{\mathrm{j2}}\right] + \frac{a_0\nu}{\pi}\left[\cos\left(\theta_{\mathrm{j}} - \varphi_{\mathrm{j1}}\right) - \cos\left(\theta_{\mathrm{j}} - \varphi_{\mathrm{j2}}\right)\right]. \quad (10)$$

Here w_1^{η} is the viscous loss and w_1^{m} is the loss in hysteresis magnetization reversal, which is numerically equal to the jump of orientational energy referred to a half-period of variation of the field.

Scheme of Numerical Solution and General Bifurcation Pattern. The equation of motion of a particle (6) is considered with the initial condition $\theta(0) = \theta_0$. Since the field intensity is equal to zero at the initial instant of time, the angle of orientation of the magnetic moment has a value of 0 or π. We set $\varphi(0) = 0$. The problem is solved by the finite-difference method on the time grid $\tau_i = i\Delta\tau$ (i = 0, 1, ...). Equation (6) is approximated by the simplest difference scheme of first order (by the Euler scheme), and the orientation of the magnetic moment at the ith instant of time will be computed by numerical minimization of the orientational energy for the values $\theta = \theta_i$ and $\tau = \tau_i$ and for the initial value $\varphi = \varphi_{i-1}$. Thus, the scheme of finding the solution has the form

$$\theta = \theta_0\,, \quad \varphi = 0\,, \quad i = 0\,;$$

$$\theta = \theta_0\,, \quad i = 1\,;$$

$$\varphi_{i-1} = \text{Minimize}\left[u\left(\theta_{i-1}, \tau_{i-1}, \varphi\right), \varphi_{i-2}\right],$$

$$\theta_i = \theta_{i-1} - a_0\,\Delta\tau\,\sin\left(\nu\tau_{i-1}\right)\sin\left(\theta_{i-1} - \varphi_{i-1}\right), \quad i = 2, 3, \ldots\,. \quad (11)$$

Two qualitatively different types of behavior that correspond to either the continuous motion of the magnetic moment or the stepwise motion may develop in the system in question depending on the values of the excitation parameters (amplitude and frequency of the field) and the initial state. Of prime interest are steady-state (stationary) motions which are also periodic in view of the periodic character of the external field. The process of the motion reaching the steady state will be controlled using the Poincaré mapping reduced in our case to a sequence of orientations of the particle and the magnetic field at the instants of time $\tau_n = nT$ (n = 0, 1, 2, ...), which are multiples of the period of variation of the field $T = 2\pi/\nu$. At these instants of time, the derivative θ, together with the field intensity, takes on the same zero value, and the direction of the magnetic moment either coincides with the selected orientation of the anisotropy axis ($\varphi = 2\pi k$, where k is an arbitrary integer) or is opposite to it ($\varphi = \pi(2k - 1)$). If, as n increases, the sequence of orientations of the particle tends to a certain limiting value

$$\theta_{\mathrm{at}}\left(\nu, a_0\right) = \lim_{n\to\infty} \theta_n\left(\nu, a_0, nT\right), \quad (12)$$

and the values of the angle φ_n correspond to one of the two possible orientations, the system reaches the stationary regime of motion. Since both directions of the anisotropy axis are physically indistinguishable, the steady-state motion

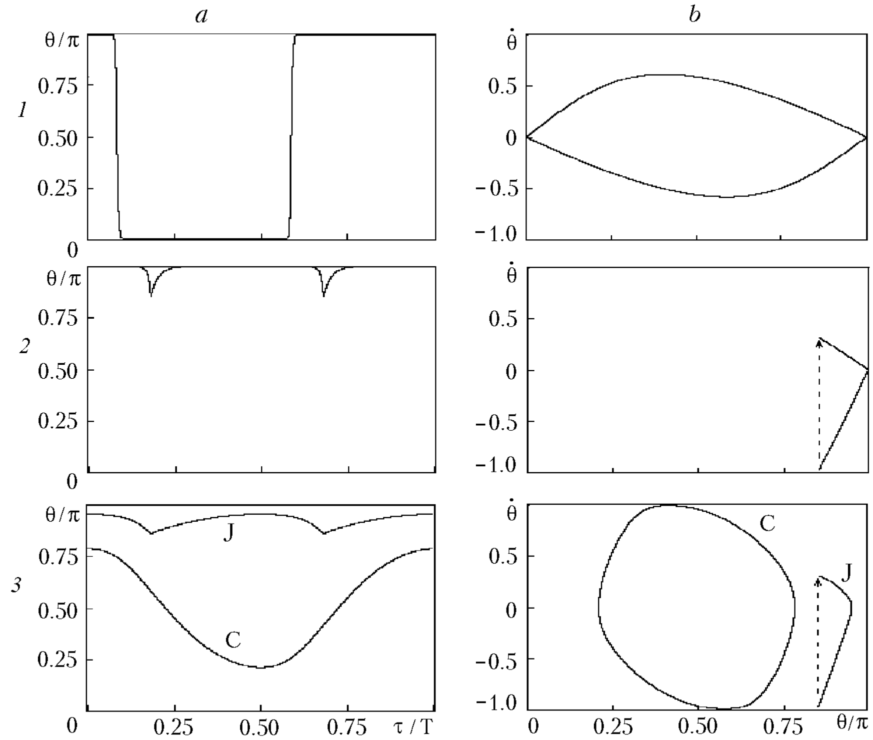


Fig. 2. Examples of the steady-state trajectories of the anisotropy axis on the physical $\theta(\tau)$ (a) and "phase" $(\theta, \dot{\theta})$ (b) planes for a fixed value of the dimensionless field amplitude (a_0 = 1.2) and at frequencies ν = 0.01 (1), 0.1 (2), and 1 (3).

with $\varphi = \pi(2k - 1)$ is identical to the motion with $\varphi = 2\pi k$ when the selected direction of the orientation vector of the particle **n** is replaced by the opposite. Thus, the quantity $\theta_{at}(\nu, a_0)$ with the supplementary condition $\varphi_n = 2\pi k$ $(n \to \infty)$ determines the stationary regime of motion for prescribed values of the amplitude and frequency of the field. Selecting θ_{at} as the initial value ($\theta_0 = \theta_{at}$), we immediately obtain the regime of stationary motion.

The important question is whether the system has one or several stationary regimes (attractors) for prescribed values of the excitation parameters. As the numerical experiments show, at different points of the excitation plane (ν, a_0), one of three situations is realized, namely, the system has either a single continuous attractor, or a single discontinuous attractor, or two attractors of different types. The sets of points of different types form, on the excitation plane, three sectors separated by the curves forming the bifurcation diagram of Fig. 1. In this figure, there is a single continuous attractor in sector I and a single discontinuous attractor in sector II; one type of motion or another is established in sector III depending on the initial orientation of the anisotropy axis. Noteworthy are certain features of the pattern obtained. The fact that we have an exceptionally continuous type of motion in the field $a_0 < 1$ is a priori clear. However, the conclusion according to which magnetization reversal of particles in a fluid matrix at field frequencies much lower than the characteristic rotational velocity of a particle (ν = 1) may follow the hysteresis type is quite unexpected.

To illustrate the mechanical behavior of a particle in different sectors of the excitation plane we give examples of the steady-state trajectories of the anisotropy axis on the physical plane $\theta(\tau)$ and on the "phase" plane $(\theta, \dot{\theta})$. (The phase space of the system is made up by the variables θ and φ. The selection made by us is more convenient and is justified by the existence of the equation $\dot{\theta} = -\sin 2\varphi$). Let us take a fixed value of the field amplitude a_0 = 1.2 and values of the frequency ν = 0.01, 0.1, and 1, arriving successively at sectors I, II, and III. The result obtained is shown in Fig. 2. Here the data in sectors I, II, and III are marked by figures 1, 2, and 3. In region III, there are two attractors — a continuous attractor (C) and a discontinuous one (J). The first attractor corresponds to the initial orientation θ_{at} = 2.475, whereas the second corresponds to θ_{at} = 3.002. We emphasize that, in the continuous regime, the particle axis oscillates with a period equal to the period of variation of the field about the mid-position $\theta = \pi/2$

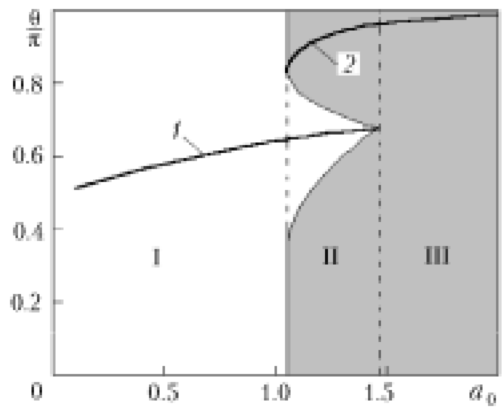


Fig. 3. Dynamic hysteresis in buildup and decrease in the dimensionless field amplitude a_0 at the frequency $\nu = 2$.

that is perpendicular to the polarization axis. In the discontinuous regime, one cycle of variation of the field includes two cycles of variation of the orientation of the anisotropy axis, and the mid-direction of the anisotropy axis is in the vicinity of the direction of polarization.

It is clear that, in sectors I and II of the excitation plane, the domains of attraction of the attractors include the entire space of initial orientations of the particle $0 \le \theta_0 \le \pi$. In sector III, each of the two attractors corresponds to its own domain of attraction. Let us consider the bifurcation behavior of the system in intersecting the boundaries of regions I, II, and III of the excitation plane. Tracking the transition from region I to region II, we find that it occurs with continuous variation in the parameter of the limiting cycle θ_{at}. This means that with a fairly slow variation in the excitation parameters at the instant of transition the system immediately turns out to be in a new stationary regime of motion. The behavior of the particle in sector III depends on from which of the adjacent sectors (I or II) the transition to this sector has been carried out. At entry from sector I, the particle preserves the regime of continuous motion in sector III to the intersection of the boundary of sector II. If we arrive at sector III from sector II, the motion remains discontinuous to the boundary of sector I. This means that the system in question possesses the property of memory and shows the effect of dynamic hysteresis.

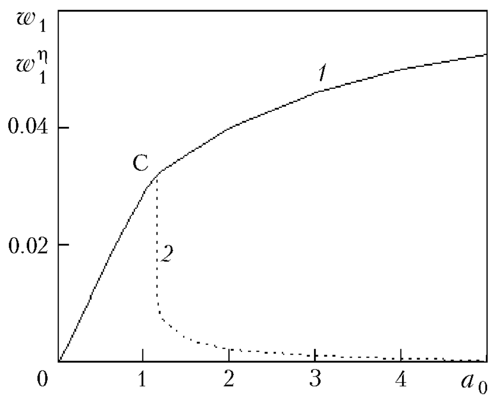
To illustrate the hysteresis effect we consider the behavior of the system with variation in the amplitude of the field with a fixed frequency $\nu = 2$. The results are presented in Fig. 3 as a diagram showing the initial orientations of the limiting cycles θ_{at} and the domains of attraction of the attractor. Here curves 1 and 2 are respectively the continuous and discontinuous attractors; the light region is the domain of attraction of the continuous attractor and the dark region is that of the discontinuous attractor; I, II, and III are the sectors determined in Fig. 1. The range of values of the field amplitude in which there are two attractors is limited by the values $a_0^{\mathrm{I,III}} = 1.05$ and $a_0^{\mathrm{II,III}} = 1.46$. Transition from the continuous regime of motion to a discontinuous one is stepwise with increase in the field amplitude to the value $a_0^{\mathrm{II,III}}$. Backward jump occurs with decrease in the field amplitude to $a_0^{\mathrm{I,III}} < a_0^{\mathrm{II,III}}$.

Energy Dissipation. To compute the total dissipation energy we apply direct summation of relation (8), assuming that δR_1 is the elementary work done on a particle over a period equal to the step of the time grid in the numerical scheme (11):

$$\delta R_1^{(i)} = IVH_0 \sin(\nu\tau_i)\,[\cos(\theta_i - \varphi_i) - \cos(\theta_{i-1} - \varphi_{i-1})]\,. \tag{13}$$

The average rate of dissipation of energy is determined by the sum of the elementary works over the period of variation of the field, divided by the value of the period. Thus, for the total dimensionless rate of dissipation of energy we have

$$w_1 = \frac{a_0\nu}{2\pi} \sum_{i=1}^{N} \sin(\nu\tau_i)\,[\cos(\theta_i - \varphi_i) - \cos(\theta_{i-1} - \varphi_{i-1})]\,. \tag{14}$$



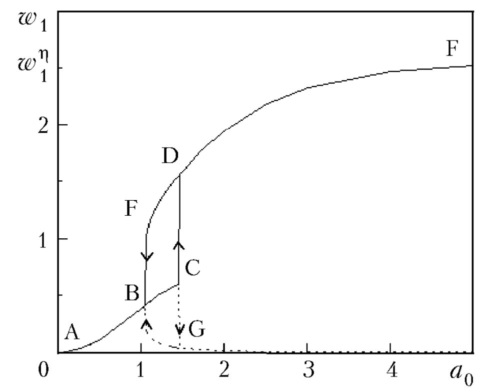


Fig. 4. Total dissipation rate (1) and its viscous component (2) vs. amplitude of the magnetic field a_0 at the frequency ν = 0.05. On the portion 0C, the total dissipation coincides with the viscous dissipation. At point C (a_0 = 1.145), the system passes from sector I of continuous motion to sector II of discontinuous motion.

Fig. 5. Hysteresis of the total (solid curves) and viscous (dashed curves) dissipation rates at the frequency ν = 2 with increase and decrease in the amplitude of the magnetic field a_0.

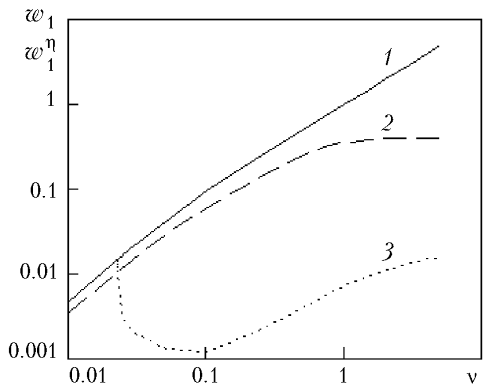
Here $N = 2\pi(\nu\Delta\tau)$ is the number of intervals of the time grid that cover one period of variation of the field. Using (10), we find the contribution of viscous dissipation according to the formula

$$w_1^\eta = N\left(\frac{\nu}{2\pi}\right)^2 \sum_{i=1}^{N} (\theta_i - \theta_{i-1})^2 . \qquad (15)$$

We emphasize that calculation from (14) and (15) must be performed after the stationary regime of motion has been reached.

The results of calculation of the total dissipation rate and its viscous component as a function of the amplitude of the magnetic field at frequencies ν = 0.05 and 2 are presented in Fig. 4 and 5. Transition from the continuous type of motion to a discontinuous one occurs at the lower of the above frequencies for the critical value $a_0^{\mathrm{I,II}}$ = 1.145. In going beyond the critical point, the viscous loss sharply decreases but the generated solid-state loss totally compensates for this decrease. The total dissipation rate changes monotonically in going beyond the critical value of the field amplitude.

In the case in question, the regime of motion varies in intersection of the bifurcation curve on the portion separating sectors I and II of the excitation plane. In the case ν = 2 (Fig. 5), there are two critical values, $a_0^{\mathrm{I,III}}$ = 1.05 and $a_0^{\mathrm{II,III}}$ = 1.46, on the line of variation of the field amplitude. Points at which two stationary regimes of motion coexist and which belong to region III of the excitation plane are between these values. The dominant influence on the system's dissipative properties is exerted in this case (see Fig. 5) by the above-described effect of dynamic hysteresis. If the field amplitude builds up (curve ACDE), dissipation is purely viscous in character up to the critical value $a_0 = a_0^{\mathrm{II,III}}$ (point C). At this point, we have a sharp drop in the viscous loss (CG), but simultaneously a magnetic hysteresis switches on, which leads to a nearly threefold stepwise rise in the total loss (CD). With decrease in the field amplitude from a certain value $a_0 > a_0^{\mathrm{II,III}}$ at the critical point $a_0 = a_0^{\mathrm{I,III}}$ (point F) the discontinuous regime of motion is replaced by a continuous one, and the total loss drops (FB). Let us consider the question on the dependence of the dissipation rate on the frequency of the field whose amplitude preserves its constant value. We note that, tracking this dependence, we move along a certain horizontal line on the excitation plane. Due to the above-described bifurcation structure of the excitation plane, there can be three fundamentally different situations, namely, the above line lies either in sector I or in sectors I and II, or intersects all three sectors. Consequently, we consider three variants: a_0 = 1, 1.4, and 2. The results of calculations for the first and third variants are shown in Fig. 6. In the case a = 1 (curve 3),



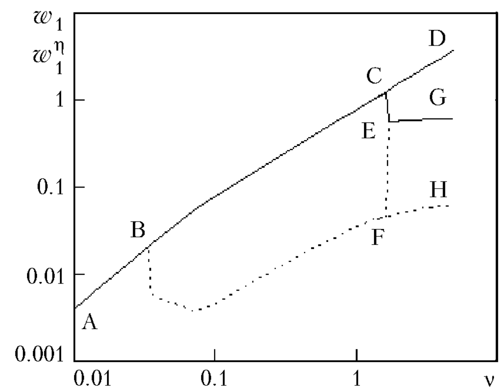


Fig. 6. Frequency dependences of the rates of total (1, 3) and viscous (2) dissipation for the field amplitude a_0 = 2 (1, 2) and 1 (3).

Fig. 7. Frequency dependences of the rates of total (solid curves) and viscous (dashed curves) dissipation for the field amplitude a_0 = 1.4.

dissipation has a purely viscous nature. The dissipation rate builds up in proportion to the frequency at low frequencies and reaches saturation ($w_1 \approx 0.41$) at $\nu > 1$. In the case a_0 = 2, at the frequency $\nu \approx 0.024$ the system passes to a discontinuous regime of motion from the continuous regime. The viscous loss sharply drops, whereas the total loss continues to build up in proportion to the field frequency. At the frequency ν = 5, the dimensionless dissipation rate is 4.87 for a_0 = 2, i.e., exceeds 12 times the limiting viscous loss (for a_0 = 1). At the frequency ν = 50, this ratio is equal to 120, etc. It is precisely the possibility of building up the solid-state loss in a liquid dispersion of ferroparticles without restriction that creates the prospects for practical use of this heat source in problems of heating.

The results of calculations for a_0 = 1.4 are given in Fig. 7. Two cases have been considered. In the first case, the frequency builds up from the zero value for a fixed amplitude. The total loss (AD curve) build up monotonically with field frequency. The viscous loss (curve ABH) to the boundary of sector II (point B, $\nu \approx 0.034$) amounts to 100% dissipation. At this frequency, the continuous regime of motion is replaced by a discontinuous one. As has already been noted, the viscous loss drops, whereas the total loss continues to build up due to magnetic hysteresis. Passage from sector II to sector III (point C) does not have any features, since, as we have seen above, the character of the particle's behavior remains constant with such a passage. Another method of realization of the process in question in which the field frequency decreases from a certain initial value is also practicable. Let this value be ν = 5 attained in the previous case. We assume that, at this frequency, the necessary value of the field amplitude has been attained by its slow increase corresponding to the movement from sector I of the excitation plane. In this case, the system is initially in the regime of continuous motion and dissipation has a purely viscous nature (point G). The dissipation rate as a function of the decreasing frequency in the process in question is shown as the curve GE in Fig. 7. The total dissipation here coincides with the viscous dissipation. Transition to a discontinuous motion occurs with decrease in the frequency to the value $\nu \approx 1.8$ when the system turns out to be in sector II of the excitation plane. The viscous dissipation drops (EF), whereas the total dissipation stepwise increases (EC). As the field frequency decreases further, the rate of total dissipation follows the curve CA and that of viscous dissipation follows the curve FBA. Thus, the regime of purely viscous dissipation for a field amplitude substantially exceeding the critical value a_0 = 1 of occurrence of magnetic hysteresis can be realized in sector III. In this case, the viscous loss appreciably exceeds the analogous loss in sector I. Thus, in the case considered the dimensionless dissipation rate on the portion EG attains a value of 0.61, which is half as much as its limiting value for a_0 = 1.

Discussion. Considering the results obtained from the viewpoint of the problem of heating, we draw a conclusion on the necessity of forming systems with physical characteristics ensuring arrival at sector II on the plane of dimensionless excitation parameters. In this connection, it is important to know the asymptotic behavior of the bifurcation curve separating sector II from sector III in the limit $\nu \to \infty$. In other words, we must find, in the limit $\nu \to \infty$, the value of the critical field amplitude for which the continuous attractor loses stability. Taking into account that the anisotropy axis oscillates about the direction $\theta = \pi/2$ in the regime of continuous motion and the oscillation amplitude tends to zero as the frequency grows ($\zeta = (\theta - \pi/2) << 1$), we rewrite Eq. (8) in the form

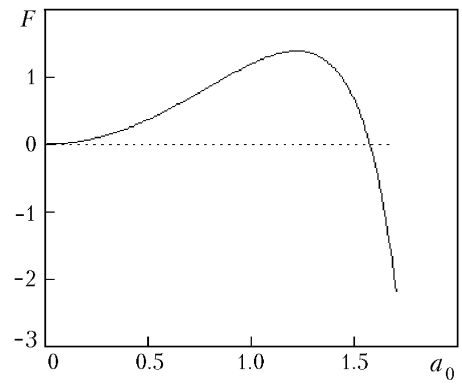


Fig. 8. Plot of the function $F(a_0)$ determining the stability ($F > 0$) or instability ($F < 0$) of the continuous attractor in the high-frequency limit.

$$\dot{\zeta} = -\, a\,(\tau)\cos(\zeta - \varphi) \approx -\, a\,(\tau)\,(\cos\varphi + \zeta\sin\varphi)\,. \tag{16}$$

Here we have discarded terms of the order ζ^2 or higher after the approximate equality sign.

Let us direct our attention to the equation of equilibrium (3), which takes the form $a(\tau)(\cos\varphi + \zeta\sin\varphi) = 2\sin\varphi\cos\varphi$ in the approximation in question. For $\zeta = 0$, its solution φ_0 is yielded by the relation $\sin\varphi_0 = a(\tau)/2$. When ζ are small, the deviation $\delta = \varphi = \varphi_0$ is also small. Leaving the terms of first order in the variables ζ and δ in Eq. (16), we reduce it to the linearized form

$$\dot{\zeta} = -\, a\,(\tau)\,[\cos\varphi_0 + (\zeta - \delta)\sin\varphi_0]\,. \tag{17}$$

Next, linearizing the equilibrium equation (3) in variables ζ and δ, we find

$$\delta = \frac{a\,(\tau)\sin\varphi_0}{a\,(\tau)\sin\varphi_0 + 2\cos 2\varphi_0}\,\zeta \equiv \frac{a^2(\tau)}{4 - a^2(\tau)}\,\zeta\,.$$

Eliminating it with the use of δ in (17), we obtain the equation of motion in the form

$$\dot{\zeta} = -\, A\,(\tau) - B\,(\tau)\,\zeta\,, \tag{18}$$

where

$$A\,(\tau) = a\,(\tau)\left(1 - \frac{a^2(\tau)}{4}\right)^{1/2}; \quad B\,(\tau) = \frac{a^2(\tau)\,[2 - a^2(\tau)]}{4 - a^2(\tau)}\,.$$

It is noteworthy that the value (averaged over the period of variation) of the function $A(\tau)$ is equal to zero and that of the function $B(\tau)$ is nonzero and is computed according to

$$\overline{B} = \frac{1}{\nu}\,F\,(a_0)\,, \quad F\,(a_0) = \int_0^{2\pi} \frac{a_0^2\sin^2 x\,[2 - a_0^2\sin^2 x]}{4 - a_0^2\sin^2 x}\,dx\,. \tag{19}$$

The second term in (18) is a small addition to the first term. Discarding it, we obtain the solution of the first approximation in the form

$$\zeta_1\,(\tau) = \zeta_0 + \frac{1}{\nu}\,P\,(\tau)\,. \tag{20}$$

Here ζ_0 is the constant of integration (with respect to the condition $\zeta_0 \ll 1$) and $P(\tau)$ is the odd periodic time function whose explicit form is not written. We only emphasize that it has an amplitude of the order of unity and a zero average value. Relation (20) describes small periodic fluctuations about the average value ζ_0. If the system is in the regime of stationary motion, we have $\zeta_0 = 0$. If the stationary motion is stable, the deviation of ζ_0 at a certain instant of time decreases after a cycle of variation of the field; otherwise it builds up. We seek the total solution of Eq. (18) by the method of variation of an arbitrary constant, setting $\zeta(\tau) = \zeta_0(\tau) + \nu^{-1}P(\tau)$. For ζ_0, Eq. (18) yields

$$\dot{\zeta}_0 = -\frac{1}{\nu} P(\tau) B(\tau) - B(\tau) \zeta_0 .$$

Let us integrate this equation with resect to time over one period of variation of the field T. Assuming that the variation in ζ_0 over the period T is small, we find the approximate relation $\Delta\zeta_0 = -\overline{B}\zeta_0 T$. It follows that the question on the stability of the continuous attractor in the high-frequency limit is solved by the sign of the quantity $\overline{B}$ (or the quantity F in (19)). The function $F(a_0)$ is plotted in Fig. 8. The asymptotic value of the field amplitude, above which the continuous regime of motion gives way to a discontinuous one, is determined from the equation $F(a_0) = 0$ and is $a_0^{\mathrm{III,II}} = 1.572$.

Conclusions. Let us give certain quantitative estimates characterizing the physical parameters of the system considered and the intensity of heat releases in it. We take, as an example, needle-shaped iron-gamma-oxide particles used in the experiments [1] on heating of a solid dispersion. The anisotropy constant for them is estimated at $K = 1.7 \cdot 10^5$ ergs/cm^3; the value of the dimensionless quantity $a = 1$ is attained in the field $H \approx 500$ Oe and the semiaxis ratio is $\lambda = 5$ ($\alpha = 48$). The characteristic time of mechanical rotation of a particle in the fluid with a viscosity η is $t^* = 1.7 \cdot 10^{-3}\eta$ (sec). The characteristic field frequency $\omega_* = 1/t^*$ corresponding to a dimensionless value of $\nu = 1$ is equal to $\omega_* = 6 \cdot 10^2 \eta^{-1}$ (sec^{-1}). It follows that in the case of a low-viscosity fluid, such as water ($\eta = 10^{-3}$ P), very high frequencies ($\sim 10^4$ Hz) are required for realization of the hysteresis regime. In a glycerin-type fluid ($\eta \sim 10$ P), this frequency decreases to ~10 Hz. According to the data obtained, the loss power in dimensionless form depends on the frequency as $w_1 \approx \nu$ for $\nu > 1$ and $a_0 > 2$. In dimensional form, the heat-release power in the suspension with a volume concentration of particles c is evaluated, under these conditions, by the relation $W = cK\omega$. For a frequency of 50 Hz and a concentration $c = 0.01$ we have $W \approx 50$ kW/m^3. Increasing the frequency by an order of magnitude, we increase the heat release to 0.5 MW/m^3.

Thus, the system considered possesses interesting dynamic and dissipative properties and enables one to ensure volume heat releases of practical significance.

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NOTATION

$a = IH/K$, dimensionless magnetic-field intensity; **B**, magnetic-field induction, G; c, volume concentration of particles; **e**, unit vector in the direction of the magnetic moment; **H**, magnetic-field intensity, Oe; **h**, unit vector in the direction of polarization of the field; H_a, intensity of the magnetic-anisotropy field, Oe; **I**, magnetization, G; K, magnetic anisotropy constant, ergs/cm^3; $\mathbf{m} = IV\mathbf{e}$, magnetic moment of a particle, G·cm^3; **n**, unit vector in the direction of the anisotropy axis; δR, elementary work, ergs; t, time, sec; t^*, characteristic time, sec; T, dimensionless period; V, volume, cm^3; U, energy, ergs; $u = U/KV$, dimensionless energy; W, heat-release power, ergs/sec; W^*, heat-release-power scale, ergs/sec; ω, dimensionless heat-release power; α, shape factor of a particle; λ, long-to-short particle semiaxis ratio; φ, angle of deviation of the magnetic moment from the anisotropy axis; φ_{j1} and φ_{j2}, angles of orientation of the magnetic moment before the jump and after it; θ, angle of deviation of the anisotropy axis from the direction of polarization; η, viscosity, P; ω, frequency, sec^{-1}; $\nu = \omega t^*$, dimensionless frequency. Subscripts: 0, amplitude value, initial value; 1, referring to a single particle; η, viscous; m, magnetic; a, anisotropy; c, critical; j, jump; at, attractor.

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